ON THE EFFICIENCY OF ASTRONOMICAL SPECTROGRAPHS

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In articles on the efficiency of spectrographs, generally very little consideration has been given to the influence of trailing the image; the present paper reviews this problem with special consideration of this fact.

Four cases can be considered:

I. It is necessary to trail the object whose spectrogram is to be taken, along the slit of the spectrograph (Small dispersion).

II. The object is kept steady over the slit, giving a spectrum of sufficient width (Medium dispersion).

III. The object is kept steady over the slit, giving a spectrum much wider than necessary (High dispersion).

IV. The object has a considerable diameter (Nebular spectrograph)

The parameters used in the present discussion are:

a) Parameters of telescope:

D_1 = Diameter of primary mirror

f = focal length of telescope

F = f/D_1 = relative aperture of telescope

E = $206265''/f$ = scale on focal plane of telescope

b) Parameters of spectrograph:

d_t = theoretical diameter of collimator beam (1)

d_v = real diameter of collimator beam

f_{ct} = theoretical focal length of collimator

f_{cv} = real focal length of collimator

F_{ct} = f_{ct}/d_t = theoretical relative aperture of collimator

F_{cv} = f_{cv}/d_v = real relative aperture of collimator

f_δ = focal length of camera perpendicular to slit (direction of dispersion or length of spectrum)

f_α = focal length of camera parallel to slit (width of spectrum)

a = aperture of slit projected on plate

h = linear aperture of slit on focal plane of telescope

h" = angular aperture of slit on focal plane of telescope

b = height of slit projected on plate

β = linear height of slit over focal plane of telescope

β" = angular height of slit over focal plane of telescope

s = diameter of stellar image

It can be easily deduced that

$$F = F_c = f/D_1 = f_{ot}/d_t \tag{1}$$

If we define

$$\text{a)}\quad m_{ot} = d_t/D_1 = \frac{f_{ot}}{f} D_1 \tag{2}$$

$$\text{b)}\quad F_{ot\delta} = f_{ot}/f$$

then

$$d_t = m_{ot} \cdot D_1 = \frac{f_{ot}}{f} D_1 \tag{3}$$

We easily obtain:

$$h = a.F_{ot\,\delta} = a \cdot \frac{f_{ot}}{f_\delta} \tag{4}$$

$$h'' = h.E \tag{5}$$

and from (2)

$$h'' = a.m_{ct} \frac{206265''}{f_\delta}$$

If for a given telescope and grating we can introduce the quantity:

$$\mu = a \cdot m_{ot} \cdot 206265'' \tag{7}$$

Then

$$h'' = \frac{\mu}{f_\delta}$$

If we fix a, D_1, f_δ and h", the necessary diameter of grating and collimator can be deduced:

$$d_t = \frac{h'' \cdot D_1 \cdot f_\delta}{a \cdot 206265''} \qquad (8)$$

As only gratings of certain diameters are produced, the only possibility left is to increase the focal length of collimator; as one of the authors has done with the DIMP (2).

Equation (8) shows that the diameter of collimator grows in linear sense with the diameter of the primary mirror and with the focal length of the spectrograph camera. This leads us to consider that there is no major gain in increasing the diameter of the primary mirror of a telescope if the theoretical collimator cannot be provided at the same time with the necessary gratings and the case may come up - for a certain camera - that the largest grating or set of gratings existing has only a very small fraction of the necessary theoretical diameter in a large telescope and, on the other hand, is quite an acceptable fraction in a smaller telescope. We therefore see that the absolute efficiency of a spectrograph may not increase if we simply enlarge the diameter of the primary telescope mirror. This problem can be made easily comprehensible comparing a telescope having a medium sized primary mirror (1 metre) with one that has a giant primary (5 metres).

In figure 1 the values of d_t (ordinate) are given in metres for these examples, giving the dispersion (hyperbolas) for the abscissa or the focal length of the camera in metres (straight lines). The enormous difference between both telescopes is clearly visible: For a dispersion of 10 Å / mm in a camera with focal length of 1,60 metres, d_t will be 0,78 m por the first and 3,88 m for the second telescope. If these values were desired for 1 Å/mm, we would have to multiply by a factor of 10, as in the equation (8), d_t is linearly dependent on f_δ. The values in the graph have been calculated taking the following values for the parameters: $a = 0{,}000\,02$ m (20 microns); $h'' = 2''$.

The width of the spectrum can be calculated with the following equation:

$$\beta'' = b \cdot m_{ct} \cdot \frac{206265''}{f_\alpha}$$

$$\beta = b \cdot F_{ct\alpha} = b \cdot \frac{f_{ct}}{f_\alpha}$$

and to calculate the slit area illuminated by the star we have

$$\sigma'' = h'' \; . \quad = a \; . \; b \; . \; . \; m_{ct}^2 \; \frac{(206265)^2}{f_\alpha \; f_\delta}$$

Let us consider the first case (small dispersion): we assume the efficiency to be proportional to the telescope diameter and inversely proportional to the factor of trailing. It can easily be deduced that the factor of trailing is inversely proportional to the telescope diameter, so that we definitely have now:

$$\varphi = \frac{h''.I}{\text{trailing}} = \frac{h''.D^2}{\text{trailing}} = h''.D^3 \text{ (as long as } h'' < s)$$

where φ expresses the efficiency. Although h'' is also inversely proportional to D, the fact that we have to trail it, signifies that h'' can have a computed value higher than the diameter of the stellar image and the efficiency is not proportional to h'' any more.

In the example, if h'' for the larger telescope is of the order of s, it will be larger for the smaller one, but we have not enough image to illuminate it, so that the equation (9) will be reduced to

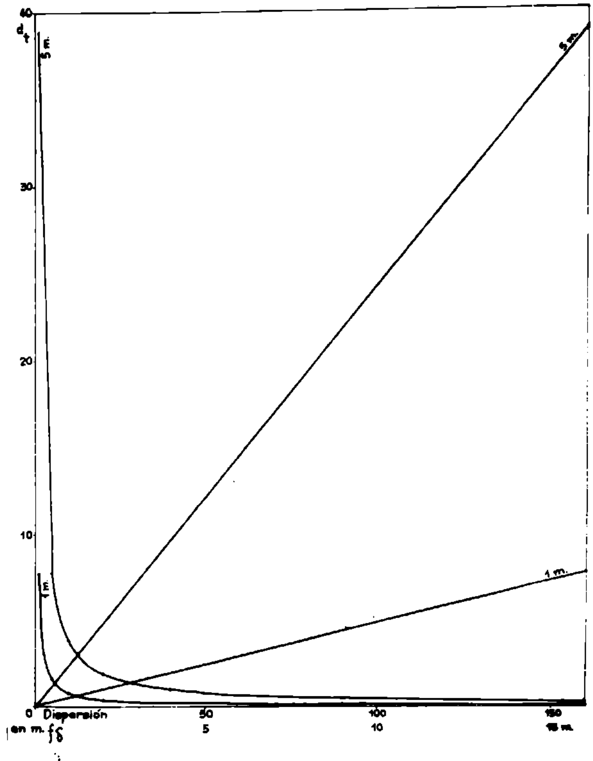
$$\varphi = D^3 \qquad \text{(as long as } h'' < s) \tag{10}$$

We shall now consider the third case (high dispersion), as the second case may be a combination of I and III, and will always be a value comprised between formulas (10) and (11). In high dispersion, the trailing is no longer necessary and h'' is inversely proportional to D; as there is always enough image to illuminate the whole slit, so that the equation (9) will be reduced to

$$\varphi = h'' \; . \; D^2 = D \tag{11}$$

The larger telescope will give a D times wider spectrum but of the same intensity as the one given by the smaller telescope, and if the smaller one gives a sufficient value of b, the larger telescope has no advantage although its construction and maintenance is more expensive.

The second case is interpolated between the first and third, as we said before; to compare the efficiency of two telescopes, formula (9) can be used, but remembering the limitation of h'' and trailing as already shown. Let us explain it with



an example:

$$\varphi_i = \frac{h''_i \cdot D_i^2}{(\text{trailing})_i} \quad ; \quad \varphi_j = \frac{h''_j \cdot D_j^2}{(\text{trailing})_j} \tag{12}$$

φ_i, φ_j represent the efficiency of the smaller and larger telescopes respectively; let us suppose that with the larger telescope no trailing of the star is necessary while with the smaller one it is. It will be seen that the increase of efficiency will be intermediate between the ratio (of diameters of their objectives) and its cube. Let us suppose the sufficient width of the spectrum b =10 a;

if the slit of the smaller h" covers an important fraction of the star diameter ($h" = \frac{1}{2} s$) the trailing necessary in the smaller telescope will be five times the diameter of the star, from which results that the efficiency of the smaller is 12,5 times smaller than that of the larger one.

Summing up, the equation (12) can be used in all cases always taking into account that $h" < s$, (otherwise the value of s has to be used instead of h") and the trailing of the telescope might not be necessary.

The case of the nebular spectrograph in IV presents the serious problem of having to reach a larger area of sky as in the stellar case; as we see by (8), d_t is directly proportional to h", so that increasing h" we get less favourable conditions than in the stellar case, which in general is compensated taking f_δ very small.

The efficiency can be modified introducing special designs or astigmatism in the cameras, in this case h" and the trailing in every special case has to be calculated, according to the characteristics of the modification introduced. In case photoelectrical scanning is employed the criteria of efficiency are different. A width of the spectrum which would be excessive for photographic plates would be advantageous for photoelectrical scanning (3).

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